A Printed Log-Periodic Koch-Dipole Array (LPKDA)

Dimitris E. Anagnostou, *Member, IEEE*, John Papapolymerou, *Senior Member, IEEE*,
Manos M. Tentzeris, *Senior Member, IEEE*, and Christos G. Christodoulou, *Fellow, IEEE*

***Abstract*—Koch-shaped dipoles are introduced for the first time in a wideband antenna design and evolve the traditional Euclidean log-periodic dipole array into the log-periodic Koch-dipole array (LPKDA). Antenna size can be reduced while maintaining its overall performance characteristics. Observations and characteristics of both antennas are discussed. Advantages and disadvantages of the proposed LPKDA are validated through a fabricated proof-of-concept prototype that exhibited approximately 12% size reduction with minimal degradation in the impedance and pattern bandwidths. This is the first application of Koch prefractal elements in a miniaturized wideband antenna design.**

***Index Terms*—Koch, log-periodic dipole antenna array, log-periodic dipole array (LPDA), log-periodic Koch-dipole array (LPKDA), miniaturized antenna.**

I. INTRODUCTION

LOG-periodic dipole array (LPDA) antennas used in radio signal detection applications can achieve high directivity and low cross-polarization ratio over a very large frequency range. Such *wideband* antennas have typically been constructed using Euclidean radiating elements [1], [2]. In applications where space and weight is restricted, antennas need to be lightweight and to have small physical size. Miniaturized concepts are often utilized.

Until today, Koch curves have been extensively and exclusively used with narrow-band antenna miniaturization schemes [3]–[6]. In the past, genetic algorithms were used as well, in order to reduce the spacing between antenna elements and, thus, the boom length [2]. In this letter, Koch-shaped dipoles [3]–[5] are introduced as the basic structural elements of a wideband log-periodic Koch-dipole array (LPKDA) for the first time, thus replacing the full-length Euclidean elements, to reduce the antenna width. The Koch prefractal shape is selected due to its wide applicability in miniaturized antennas, described extensively in [3]–[6]. The shortening scheme does not necessarily need to be based on prefractal shapes. However, due to the nonopposing directions of the current on each dipole, the Koch shape is expected to have one of the least deteriorating effects on both the co-polarized and the cross-polarized patterns.

TABLE I
SINGLE ELEMENT MINIATURIZATION (EUCLIDEAN VERSUS KOCH)

Antenna Type	Resonant Freq. (GHz)	Total Length (mm)	Vertical Length (mm)	BW (%)	Length Reduction (%)
Euclidean	1.82	28.4	28.4	13	
Koch	1.82	30.3	25.6	12.6	
Vertical Length % Reduction of K1 Koch vs. Euclidean:					*9.86%*
Vertical Length % Reduction of K2 Koch vs. Euclidean:					*15.07%*
Vertical Length % Reduction of K3 Koch vs. Euclidean:					*17.15%*

The main goals of this letter are: *a*) to apply a prefractal curve in a wideband antenna design, and *b*) to develop and characterize a proof-of-concept prototype that validates the proposed concept. Design considerations are presented and the antenna performance is discussed. Once the concept is shown, other miniaturization schemes mentioned in [5] for narrowband antennas may also be used, giving more potential value to this letter.

II. LPKDA ANTENNA DESIGN

An LPKDA design is created by substituting all Euclidean dipoles of a conventional (Euclidean) LPDA, with Koch dipoles. Euclidean elements are straight wires or planar strips, often noted in literature as “K0” (Koch curves of 0th iteration). Koch wire shapes are obtained by dividing a wire of length l into three segments of length $l/3$ and replacing the middle segment by two segments of length $l/3$ intersecting at a 60° angle. This way a first iteration (K1) Koch curve is obtained. By repeating this procedure for each segment more iterations (K2, K3, and so on) can be obtained. In this letter, K1 curves were used to replace the straight wires of an LPDA. The angle of intersection was modified to 45° in order to maintain low levels of cross-polarized radiation.

This antenna is designed to cover the 2–3 GHz frequency range. It is milled on a $t = 1.588$ mm thick FR-4 laminate with relative permittivity $\varepsilon_r = 4$. The length of the largest element is determined by the lowest frequency of operation. Initially, single-element Euclidean and Koch-shaped monopoles at 2 GHz were simulated. As shown in Table I, a Euclidean element with length $L_{\mathrm{Eucl}} = 28.4$ mm, resonates at 1.82 GHz. The vertical length (assuming the orientation of Fig. 1) of a Koch element to resonate at the same frequency was found to be $L_n = 25.6$ mm. This indicates that a vertical-size reduction: $((L_{\mathrm{Eucl}} - L_n)/L_{\mathrm{Eucl}}) \cdot 100\% = 9.86\%$ can be obtained using the first-iteration Koch shape.

Manuscript received May 03, 2008; revised June 04, 2008. First published June 18, 2008; current version published December 12, 2008. This work was supported in part by the National Science Foundation/EPSCoR Grant 0554609.

D. E. Anagnostou is with the Electrical Engineering Department, South Dakota School of Mines and Technology, Rapid City, SD 57701 USA (e-mail: danagn@ieee.org).

J. Papapolymerou and M. M. Tentzeris are with the School of Electrical and Computer Engineering, Georgia Institute of Technology, Atlanta, GA 30308 USA (e-mail: papapol@ece.gatech.edu; etentze@ece.gatech.edu).

C. G. Christodoulou is with the Electrical and Computer Engineering Department, University of New Mexico, Albuquerque, NM 87106 USA (e-mail: christos@ece.unm.edu).

Color versions of one or more of the figures in this paper are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/LAWP.2008.2001765

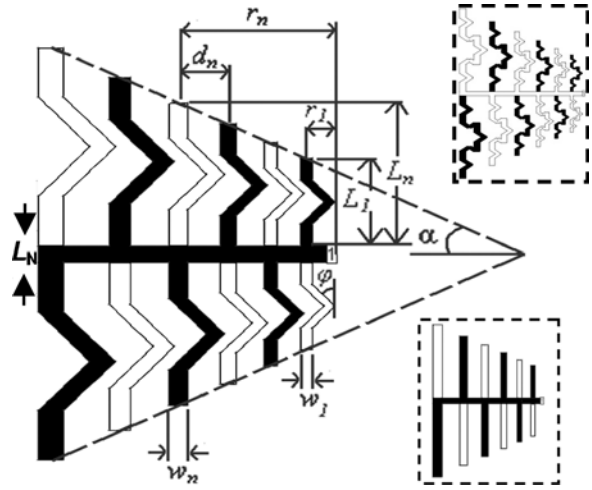

Fig. 1. Design parameters of the LPKDA antenna. The bottom right inset shows a Euclidean LPDA to illustrate the differences between the two designs. The top right inset shows a 2-iteration LPKDA $(\mathrm{LPK^2DA})$.

All dipole elements are sized proportionally using the method described by Campbell [7] for planar LPDA antennas. The LPKDA is expected to perform very similarly to the Euclidean LPDA, but to occupy a smaller area of the circuit board. The area occupied by a planar LPKDA is: $A_{\mathrm{Koch}} = h(L_1 + D + L_N)$, where L_1 and L_{N} are the vertical lengths of the smallest and largest monopoles respectively and D is the width of the feedline. Assuming the boom length, h, remains constant and $D \ll L_n$ for all elements, the area ratio reduces to: $R = A_{\mathrm{Koch}}/A_{\mathrm{Eucl}} = 0.901$.

Extending the concept to higher iterations, it can be found that a 2-iteration (K2) monopole would need to have length $L_{\mathrm{n2}} = 24.12$ mm, and would achieve 15.07% length reduction. Also, a 3rd iteration curve (K3) would need to be only $L_{\mathrm{n3}} = 23.53$ mm long resulting in 17.15% reduction. As the iteration order increases, the fractal feature length becomes smaller and the effect on antenna miniaturization becomes less significant. Thus, applicability of this concept is limited to the first three or four iterations, achieving a size reduction of up to ~18%. This is expected for prefractal Koch-shaped miniaturized antennas [4], [5], as well as other antennas with similar radiation and miniaturization mechanisms.

III. SIMULATION RESULTS

A proof of concept LPKDA prototype was compared to a Euclidean LPDA with directivity $D = 6.5$ dBi, based on the corrected Carrel's tables [8], [9]. The directivity value determines the design parameters (spacing factor σ, and scaling factor τ). Using the notation of Fig. 1, $\sigma = ((1-\tau)/4\tan\alpha) = (d_n/4L_n) = 0.076$, where $\tau = (L_n/L_{n+1}) = (r_n/r_{n+1}) = 0.85$ and the antenna's half-angle is $a = 26^\circ$.

The exact same values for σ and τ were used also in the LPKDA design, in order to compare the performance of antennas that: *a)* have been designed using the same methodology, *b)* maintain their log-periodicity, *c)* are expected to have similar performance, yet, should occupy different areas.

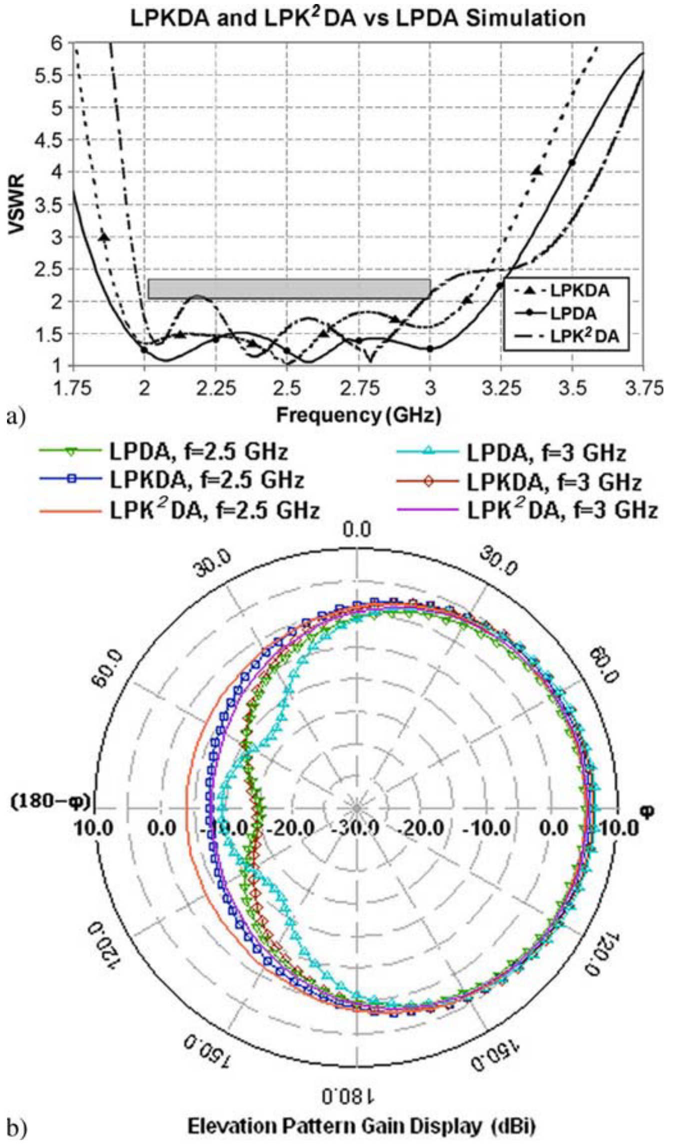

Fig. 2. Comparison between the Euclidean and the Koch-element LPDA antennas with respect to their a) VSWR (the 2–3 GHz band is shown) and b) radiation pattern at 2.5 and 3 GHz.

Simulations were conducted using IE3D and the input impedance of the zero, first and second iteration LPDAs is shown in Fig. 2(a), and the $\mathrm{VSWR} < 2$ frequency range is from 1.9 to 3.2 GHz. The simulated gain patterns at 2.5 and 3 GHz [Fig. 2(b)] show that both antennas radiate in a directive and similar to each other way. The relatively large beamwidth is expected because of the small number of elements used to achieve such large bandwidth.

The directivity of the LPKDA antenna is $D_{\mathrm{LPKDA}} = 6.2$ dBi, only 0.3 dB lower than the theoretical value for LPDAs given by Carrel's tables [8], [9]. These results show that the LPDA and LPKDA antennas have very similar performance and can cover the same range of frequencies with directive patterns. In addition, the total area of the LPDA is 1505 mm^2, while the LPKDA's is 1323 mm^2, miniaturized by 12.1%.

To investigate the effect of the second iteration Koch elements on the antenna's miniaturization, additional simulations

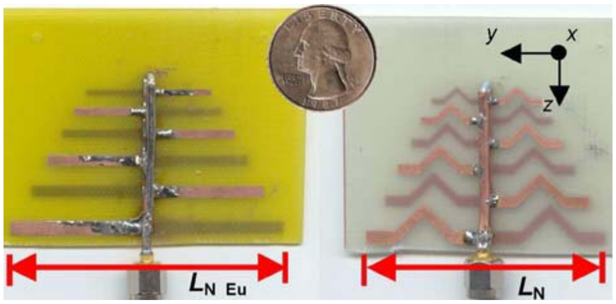

Fig. 3. Photo of the fabricated LPDA (left) and LPKDA (right) antennas. The shadowed strips are printed in the backside of the substrate.

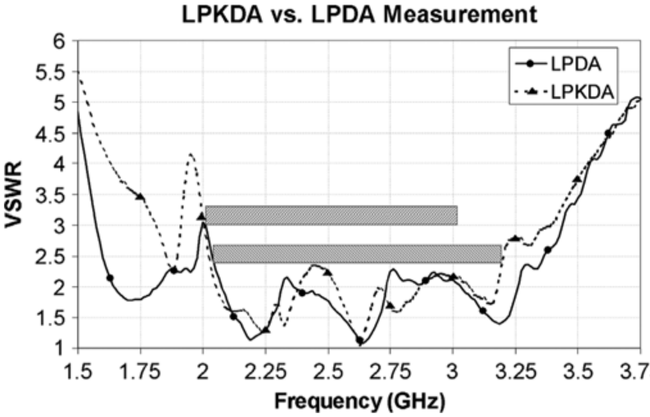

Fig. 4. Measured VSWR of the LPDA and LPKDA antennas. Both have similar response and covered the 2–3 GHz band with a $\mathrm{VSWR} < 3$, and the 2.05–3.2 GHz range with a $\mathrm{VSWR} < 2.35$.

of the antenna shown on the top-right inset of Fig. 1, which is a 2-iteration Koch element log periodic antenna $(\mathrm{LPK^2DA})$, were performed. For a 2 GHz lowest frequency of operation, its total surface was 1202 mm^2, which is 8.03% smaller than the LPKDA, and 20.1% than the Euclidean LPDA. The miniaturization trend of the log-periodic structure shows that Koch LPDAs can achieve greater miniaturization than their single monopole counterparts. This result is consistent with the fact that as the fractal iteration increases, the effect on antenna size becomes less significant (e.g., 12% from Euclidean to first iteration, but only 8.03% from first to second iteration). A similar trend is expected for higher iteration antennas, which, however, required more than 2 GB of computer memory, as they are complex, electrically large structures with fine details, and were not simulated.

On the other hand, the increased complexity of the structure seems to cause a faster deterioration of the antenna performance. The second iteration $\mathrm{LPK^2DA}$ pattern is shown in Fig. 2(b). It has smaller average gain (approximately 5.2 dBi) and smaller front-to-back ratio, caused by the increased number of current components with opposite directions on the structure. In addition, its bandwidth barely covers the 2–3 GHz range. The higher Q of the Koch elements with iteration larger than one, becomes a predominant limitation factor on the antenna's bandwidth, and overpowers the enhancement obtained by the log-periodicity of the geometry.

IV. EXPERIMENTAL CHARACTERIZATION

To investigate the validity of the simulations, an LPDA and a first iteration LPKDA prototype were milled and measured. The second or higher iteration LPKDAs have some very fine feature sizes and sharp angles and photolithography would be a more suitable fabrication method. The concept principles though are clearly illustrated from the measurements of the first iteration prototype and the traditional LPDA.

The fabricated antennas are shown in Fig. 3. They are both fed through their wider edge, using two coaxial cables that are manually soldered along their feedline until their narrow end, which is the feed connection point. This coaxial infinite balun provides the necessary wideband matching and the appropriate current phase to each dipole. At the same time, this feeding method is easier to implement than previously published planar LPDAs that used striplines [7].

Both antennas have approximately equal boom length, and miniaturization is mainly achieved because of the reduced width of the structure measured along the y-axis (Fig. 3). The measured VSWR of both antennas is very similar (Fig. 4), and the bandwidth from 2.05–3.2 GHz was realized with a $\mathrm{VSWR} < 2.35$. Still, a perfect match was not achieved and the reasons for that are analyzed below for the benefit of the antenna engineers.

There is a difference between the simulated and fabricated excitation port. For simulation purposes, the most appropriate feed is a differential port placed at the narrow (radiating) end of the antenna [11]. This port provides the necessary wideband matching but is also an "ideal" small port that does not incorporate the geometry of an SMA connector and also does not affect the radiation patterns. In practice, wideband matching can be achieved with an "infinite" balun. This is constructed by manually soldering two coaxial cables on the top and bottom of the substrate. One cable (i.e., top) carries the RF signal while the other (i.e., bottom) is a mirror cable used for symmetry. This balun is extremely hard to model in simulations. However, both cables load the feed line. In addition an SMA connector is connected at the wider edge of the antenna for measurement purposes. The soldering of the cables and SMA is critical, and any imperfections affect the wideband matching performance of the balun. By using automated soldering for the SMA and infinite balun connections, the matching performance can be improved.

The small number of elements—used here as a proof of concept also affect the matching. LPDAs typically have 6.5 dBi $< D <$ 11 dBi, and here a compact design with $D = 6.5$ dBi was chosen. For compact LPDAs, larger values of α (smaller τ) are used that result in designs with smaller number of dipole elements and, thus, fewer elements in the antenna's active region. This results in a less smooth transition between the elements of the active and inactive regions, causing larger variations in the input impedance [10], which is what can also be observed in Fig. 4. By adding more elements to the LPDA (i.e., loading the feedline), both the bandwidth and the matching can be improved. Measurements of a 12-element planar LPDA [12] verified the above by exhibiting a $\mathrm{VSWR} < 2$. In addition, if modified fractal shapes are considered, matching can be improved by slightly rounding the sharp corners of the antenna, to smoothen the current's propagation from one bend to another.

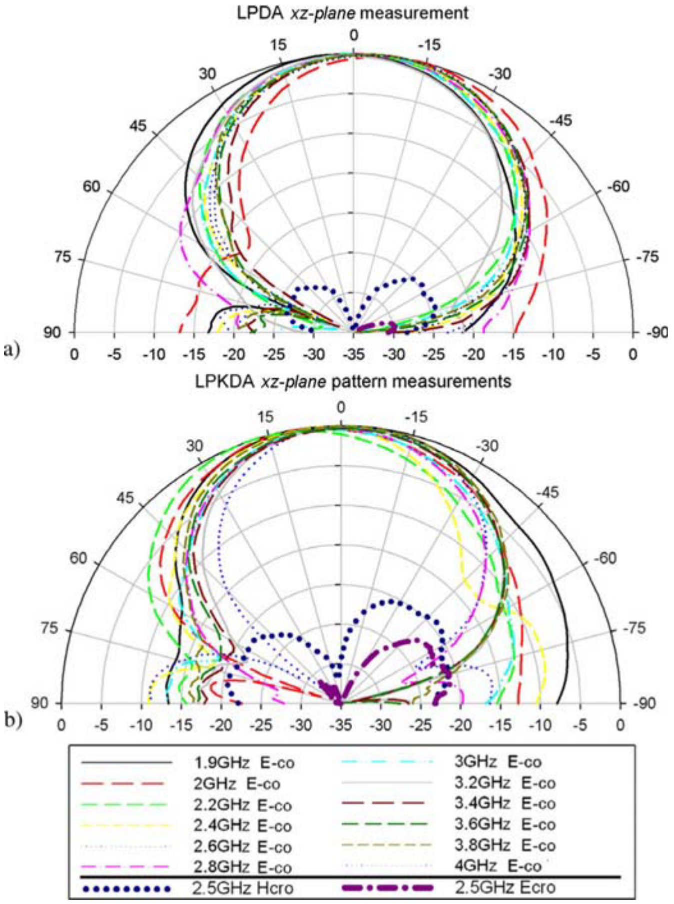

Fig. 5. Measured normalized radiation patterns of: a) LPDA antenna and b) LPKDA antenna, in the $x-z$ plane from 1.9 GHz to 4.0 GHz.

These log-periodic antenna designs are scalable, meaning that different directivity and bandwidth can easily be obtained by varying the σ and τ [8], [9]. So, for larger bandwidth or higher gain applications with space limitations, this antenna is a good candidate.

The measured patterns of both antennas are very similar and directive as shown in Fig. 5(a) and (b). The main beam is in the endfire direction from 2 GHz up to more than 3 GHz. Sidelobe levels range from -10 dBi to less than -30 dBi as expected for this type of antennas. A better comparison of the measured results is shown in Fig. 6(a) and (b), where patterns of both antennas at different frequencies are superimposed. The $x-y$ plane pattern is symmetric and is shown only to illustrate that all elements are fed appropriately and with equal current magnitude.

The use of Koch elements prompts for higher levels of cross-polarization, originating from the component of the electric current density that is vertical to the Koch dipole axis (i.e., in the z-direction). In addition, the planar nature of the antenna elements (and, thus, of their surface current density) shall excite cross-polarized radiation. For characterization purposes, such measurements at 2.5 GHz for both antennas were conducted and the results are shown in Fig. 5(a) and (b). The Euclidean LPDA has a maximum cross-polarization level lower than -24 dBi [Fig. 5(a)]. The LPKDA has stronger cross-polarization with maximum value approximately -19 dBi [Fig. 5(b)], matching well with the simulated result, which was below -22 dBi (not shown here to maintain plot clarity).

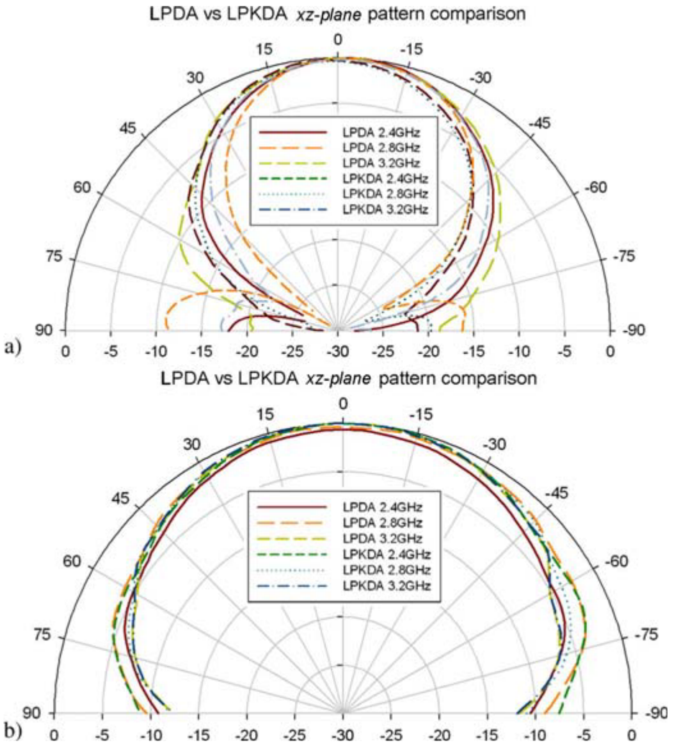

Fig. 6. Measured normalized radiation patterns of both antennas in the a) $x-z$ plane and b) $x-y$ plane. The $x-y$ plane pattern is symmetric, indicating that all elements are fed with symmetric current amplitude.

The strongest cross-polarized field component of the LPKDA is the H_{cro}, which is more intense at nonendfire directions. The measurement shows a cross-polarization discrimination level of 19 dB or lower at ±45°, which is outside of the antenna's half power beamwidth. This angle is equal to the geometry's φ angle, chosen to be 45° instead of 60° to maintain any cross-polarization at low levels.

It is important to note that the radiation pattern maximum is at boresight and cross-polarized radiation in that direction is minimal, if not null [Fig. 5(b)]. More precisely, the cross-polarization discrimination level increases to 30 dB or higher as one approaches the boresight, which is the direction in which such antennas are most often used as receivers.

Cross-polarization values lower than -40 dB for such antennas might be unrealizable due to their planar structure. Yet, their ease of fabrication makes them an appealing, lowcost and easy to integrate antenna solution.

V. Discussion and Conclusions

Koch prefractal curves were used for the first time in the miniaturization of *wideband* antennas, such as the log-periodic Koch-dipole antenna (LPKDA) presented in this letter. Here, as an example design, an antenna with the least number of elements was used to cover the specified (and largest possible for this number of elements) bandwidth, from 2–3 GHz. The antenna performance is comparable to a traditional LPDA and covers the design frequency range with similar and directive patterns of constant gain, while occupying an area reduced by more than

12%. This can also lead to reduced weight and materials used, especially in mass production receiving antenna applications.

This size miniaturization was achieved with the use of the simplest form (i.e., first iteration) of Koch curves. Extending a previous letter by Best [5], an increased number of iterations of the narrowband Koch monopoles shall lead to larger size reductions. This trend was affirmed here for wideband antennas as well, through simulations of up to a second iteration LPKDA that can result in approximately $\sim$18%–20% size reduction. Miniaturization is expected to progress until saturation, which usually occurs after the third or fourth iteration. Additional iterations provide little benefits when compared to the design effort and to the printed circuit board fabrication accuracy.

As with most antennas, miniaturization comes at an expense. Here, this expense is reflected upon *a*) a slightly higher cross-polarization field component, mostly at nonboresight directions, *b*) a higher VSWR which is enhanced by the loading of the feedline and the few elements in the antenna's active region, and *c*) a gain reduced by 0.3 dBi due to the slightly larger beamwidth and the increased VSWR. The miniaturization concept of such antennas, however, was shown and can be extended to more wideband and/or directive designs using more radiating elements. Such designs will have similar levels of cross-polarization (i.e., around -20 dB or less) and always very low at boresight (around -30 dB or less).

Even with these shortcomings, the miniaturized LPKDA performs well and covers the entire frequency range with constant gain, as expected from a conventional LPDA. This concept enables investigations on more compact LPDAs. Meander and "zig-zag" dipoles [5] that fill more efficiently the antenna volume are candidate elements that give potential to this letter.

The structure of the LPKDA is planar and relatively simple to fabricate using standard PCB fabrication techniques. It can also be directly integrated with planar microwave circuits. The design concept can easily be scaled for applications with different bandwidth and/or directivity requirements by adjusting the angle α and by adding more elements, which shall also improve the VSWR.

ACKNOWLEDGMENT

The authors would like to thank Prof. C. Jerald Buchenauer from the University of New Mexico for his insightful suggestions and discussions in planar log-periodic dipole arrays.

REFERENCES

[1] H. Gemmeke, "Radio detection of ultra high-energy cosmic ray air showers," in *Proc. Nuclear Science Symp. Conf. Rec.*, Oct. 23–29, 2005, vol. 2, pp. 1080–1083.

[2] S. El-Khamy, M. Mangoud, M. Aboul-Dahab, and A. Zaki, "Fractal multiband antennas using GA/MOM optimized log periodic dipole rrays," in *Proc. IEEE APS Int. Symp.*, Jun. 20–25, 2004, vol. 4, pp. 3433–3436.

[3] C. P. Baliarda, J. Romeu, and A. Cardama, "The Koch monopole: A small fractal antenna," *IEEE Trans. Antennas Propagat.*, vol. 48, no. 11, pp. 1773–1781, Nov. 2000.

[4] K. J. Vinoy, J. K. Abraham, and V. K. Varadan, "On the relationship between fractal dimension and the performance of multi-resonant dipole antenna using Koch curve," *IEEE Trans. Antennas Propagat.*, vol. 51, no. 9, pp. 2296–2303, Sep. 2003.

[5] S. R. Best, "On the resonant properties of the Koch fractal and other bent wire monopole antennas," *IEEE Antennas Wireless Propagat. Lett.*, vol. 1, no. 1, pp. 74–76, Jan. 2002.

[6] A. Sundaram, M. Maddela, and R. Ramadoss, "Koch-fractal folded-slot antenna characteristics," *IEEE Antennas Wireless Propagat. Lett.*, vol. 6, pp. 219–222, 2007.

[7] R. L. Carrel, "The design of log periodic dipole antenna," in *Proc. IRE Int. Convention Rec.*, 1961, pp. 61–75, pt. 1.

[8] P. C. Butson and G. T. Thompson, "A note on the calculation of the gain of log-periodic dipole antennas," *IEEE Trans. Antennas Propagat.*, vol. 24, no. 1, pp. 105–106, Jan. 1976.

[9] C. K. Campbell, I. Traboulay, M. S. Suthers, and H. Kneve, "Design of a stripline log-periodic dipole antenna," *IEEE Trans. Antennas Propagat.*, vol. 25, no. 5, pp. 718–721, Sep. 1977.

[10] Personal Communication With Dr. J. Zheng of Zeland Software, Inc.

[11] C. A. Balanis*, Antenna Theory: Analysis and Design*, 3rd ed. New York: Wiley, 2005.

[12] A. A. Gheethan and D. E. Anagnostou, "Reduced size Planar Log-Periodic Dipole Arrays (LPDAs) using rectangular meander line elements," presented at the IEEE APS Int. Symp., San Diego, CA, Jul. 2008.